

Evidence for mass dependent effects in the spin structure of baryons

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Abstract

We analyze the axial-vector form factors of the nucleon hyperon system in a model with mass dependent quark spin polarizations. This mass dependence is deduced from an earlier analysis [1,2] of magnetic moment data, and implies that the spin contributions from the quarks to a baryon decrease with the mass of the baryon. When applied to the axial-vector form factors, these mass dependent spin polarizations bring the various sum-rules from the model in better agreement with experimental data. Our analysis leads to a reduced value for the total spin polarization of the proton.

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I. INTRODUCTION

In two earlier papers [1,2] we have discussed the magnetic moments of baryons in an extension of the quark model, which allows for general flavor symmetry breaking and where the quark magnetic moments are allowed to vary with the isomultiplet B . The magnetic moments of the baryons in this model can be written as a linear sum of contributions from the various flavors

$$\mu(B^i) = \mu_u^B \Delta u^{B^i} + \mu_d^B \Delta d^{B^i} + \mu_s^B \Delta s^{B^i}, \quad (1)$$

where μ_f^B is an effective magnetic moment of the quark of flavor f in the isomultiplet B and Δf^{B^i} is the corresponding spin polarization for baryon B^i , i being the baryon charge state. By symmetry arguments the Δf^{B^i} 's in the octet baryons can be expressed as constant linear combinations of the three Δf 's for the proton, which are the only spin polarizations needed to describe the octet:

$$\Delta f^{B^i} = \sum_{f'} M(B^i)_{ff'} \Delta f', \quad (2)$$

where f, f' runs over u, d, s , and the $M(B^i)$'s are matrices with constant elements. In particular for the six mirror symmetric baryons of type $B(xyy)$, where x and y are different flavors, we have $\Delta y^{B^i} = \Delta u$, $\Delta x^{B^i} = \Delta d$ and $\Delta z^{B^i} = \Delta s$ where the flavor z is the non-valence quark flavor. In the non-relativistic quark model (NQM) the values of these spin polarizations are $\Delta u = \frac{4}{3}$, $\Delta d = -\frac{1}{3}$ and $\Delta s = 0$.

Due to the homogeneity of the right hand side of (1), it is a question of definition if the dependence on the baryon multiplet is considered to be associated with the quark magnetic moment rather than with the spin polarization. In Refs. [1] and [2] we have chosen to analyze the data by keeping the spin polarizations fixed throughout.

Here we will analyze the opposite situation, where the spin polarization is instead assumed to vary with the baryon multiplet and the quark magnetic moments are the same for all multiplets.

This scheme has the advantage of making the properties of the quarks static and environment independent. Since the effective magnetic moment of a quark in the NQM has the form

$$\mu_f = \frac{e_f}{2m_f}, \quad (3)$$

e_f being the quark charge, this means that there is no dependence of the effective quark mass m_f on B . This is in accordance with the fact that the same constituent quark masses can be used successfully to predict the baryon octet and decuplet masses with only a hyperfine splitting interaction in the Hamiltonian. The disadvantage is that the spin structure varies from multiplet to multiplet.

The most important further merit of this interpretation, and the one that we are going to analyze here, is that the sum-rules governing the axial-vector form factors are better fulfilled in this scheme, although the errors are still somewhat large to definitely decide between either of the two ways of attributing the mass dependence effect.

II. CALCULATING THE MODEL PARAMETERS

The spin structure parameters in the expressions for the magnetic moments and in the deep inelastic scattering experiments and axial-vector form factors are not a priori the same. In many models they are nevertheless proportional [3], and can be normalized to be the same. We normalize them to the axial-vector form factor $g_A^{np} = 1.2573$, as is generally done.

We write the baryon magnetic moments as

$$\mu(B^i) = \sum_{f,f'} \mu_f \alpha(B) M(B^i)_{ff'} \Delta f', \quad (4)$$

where μ_f is the magnetic moment of the quark of flavor f and Δf is the corresponding spin polarization. The factor $\alpha(B)$ is an overall factor, the same for all flavors, depending only on the isomultiplet B . The flavor symmetry breaking is then accounted for by the quark magnetic moments that are free parameters. This symmetry breaking is assumed to be the same for all isomultiplets.

In our previous analysis we associated the factor $\alpha(B)$ with the quark magnetic moments and defined $\mu_f^B = \alpha(B)\mu_f$ as in equation (1). We can choose to normalize $\alpha(B)$ to $\alpha(N) = 1$, in which case $\mu_f^N = \mu_f$. The other values of α can then be obtained from the previously extracted values of the μ_d^B 's as [1,2]

$$\alpha(\Lambda) = \mu_d^\Lambda / \mu_d^N = 0.88 \pm 0.04, \quad (5)$$

$$\alpha(\Sigma) = \mu_d^\Sigma / \mu_d^N = 0.91 \pm 0.01, \quad (6)$$

$$\alpha(\Xi) = \mu_d^\Xi / \mu_d^N = 0.85 \pm 0.03. \quad (7)$$

We will now instead associate $\alpha(B)$ with the spin polarizations. Equation (2) is then rewritten as

$$\Delta f^{B^i} = \sum_{f'} M(B^i)_{ff'} \alpha(B) \Delta f'. \quad (8)$$

Thus, *e.g.* in the mirror symmetric baryons $B(xyy)$, we instead have $\Delta y^{B^i} = \alpha(B)\Delta u$, $\Delta x^{B^i} = \alpha(B)\Delta d$ and $\Delta z^{B^i} = \alpha(B)\Delta s$.

The values of $\alpha(B)$ can be well fitted to a linear function of the mean mass of B as shown in Fig. 1. The linear relation is

$$\alpha(m) = 1 - 0.376(m - 0.939), \quad (9)$$

when m is expressed in GeV. We will continue to use $\alpha(B) \equiv \alpha(m_B)$ in the following.

If this relation is extrapolated to the decuplet resonances, it can be tested by measuring some of their magnetic moments. It is then possible to fit the expression for $\mu(\Omega^-)$ to obtain the value of $\alpha(\Omega^-)$ [2].

The most remarkable property of this fit is that the quark spin polarization of a baryon, and thus also the contribution from its quark magnetic moment, vanishes at $m \approx 3.6$ GeV, provided the linear relation does not break down before we reach this value.

We will nevertheless test this linear relation in what follows by using the interpolated α 's from the equation above. These values are

$$\alpha(\Lambda) = 0.93 \pm 0.02, \quad (10)$$

$$\alpha(\Sigma) = 0.90 \pm 0.02, \quad (11)$$

$$\alpha(\Xi) = 0.86 \pm 0.02. \quad (12)$$

To illustrate why this B dependent factor is needed we regard the sum-rule

$$\mu(p) + \mu(\Xi^0) + \mu(\Sigma^-) - \mu(n) - \mu(\Xi^-) - \mu(\Sigma^+) = 0, \quad (13)$$

which follows when the quark magnetic moments and spin polarizations both are independent of B . It is badly broken by the experimental data so that the left hand side is instead 0.49 ± 0.05 . In our more general parameterization this sum-rule becomes

$$\mu(p) + \frac{\mu(\Xi^0)}{\alpha(\Xi)} + \frac{\mu(\Sigma^-)}{\alpha(\Sigma)} - \mu(n) - \frac{\mu(\Xi^-)}{\alpha(\Xi)} - \frac{\mu(\Sigma^+)}{\alpha(\Sigma)} = 0. \quad (14)$$

Due to the construction of the α 's this sum-rule is satisfied.

III. THE AXIAL-VECTOR FORM FACTORS

Whether we associate the $\alpha(B)$ factors to the quark magnetic moments or the spin polarizations, obviously does not affect the analysis of the magnetic moments. However, the analysis of the axial-vector form factors will be modified when we let the spin polarizations be given by (8).

The axial-vector form factors can in this parameterization now be written

$$g_A^{np} = \Delta u - \Delta d, \quad (15)$$

$$g_A^{\Lambda p} = \frac{1}{3}(2\Delta u - \Delta d - \Delta s)\alpha(\Lambda), \quad (16)$$

$$g_A^{\Xi\Lambda} = \frac{1}{3}(\Delta u + \Delta d - 2\Delta s)\alpha(\Xi), \quad (17)$$

$$g_A^{\Sigma n} = (\Delta d - \Delta s)\alpha(\Sigma). \quad (18)$$

This can be used to derive the two sum-rules

$$\frac{g_A^{\Xi\Lambda}}{\alpha(\Xi)} + \frac{g_A^{\Lambda p}}{\alpha(\Lambda)} = \frac{g_A^{\Sigma n}}{\alpha(\Sigma)} + g_A^{np}, \quad (19)$$

$$\frac{g_A^{\Xi\Lambda}}{\alpha(\Xi)} + g_A^{np} = 2\frac{g_A^{\Lambda p}}{\alpha(\Lambda)}, \quad (20)$$

which are barely satisfied without the α 's. The relations are satisfied as follows

$$(0.98 \pm 0.07) \quad 1.07 \pm 0.06 = 1.04 \pm 0.09 \quad (1.06 \pm 0.08), \quad (21)$$

$$(1.51 \pm 0.05) \quad 1.55 \pm 0.06 = 1.57 \pm 0.05 \quad (1.46 \pm 0.03), \quad (22)$$

corresponding to the two equations above. The numbers in parentheses are the values without the α 's (i.e. $\alpha \equiv 1$). The experimental values $g_A^{np} = 1.2573 \pm 0.0028$ and $g_A^{\Xi\Lambda} =$

0.25 ± 0.05 are taken from the Review of Particle Properties data table [4], the value $g_A^{\Sigma n} = -0.20 \pm 0.08$ from Hsueh *et al.* [5] and the value $g_A^{\Lambda p} = 0.731 \pm 0.016$ from Dworkin *et al.* [6]. For a more detailed discussion we refer the reader to Sec. 3 of Ref. [1].

Although the improvement relative to the case without the α 's might not be dramatic, both sum-rules are definitely better satisfied with the α 's.

As a further test we calculate the constant $R = \frac{\Delta u - \Delta d}{\Delta u - \Delta s}$ defined in Ref. [1]. This constant has the value $R = 1.18 \pm 0.01$ from the magnetic moment data. Our expression for this constant, expressed in terms of axial-vector form factors, is now

$$R = \frac{2g_A^{np}}{g_A^{\Lambda p}/\alpha(\Lambda) + g_A^{\Xi\Lambda}/\alpha(\Xi) + g_A^{\Sigma n}/\alpha(\Sigma) + g_A^{np}} = 1.19 \pm 0.06. \quad (23)$$

This is again an improvement over the value $R = 1.23 \pm 0.06$ found in Ref. [1].

The four axial-vector form factors can be parameterized by two variables, which we choose as $\Delta u - \Delta d = g_A^{np}$ and $a_8 = \Delta u + \Delta d - 2\Delta s$. Another often used parameterization is $F + D = \Delta u - \Delta d$, $F/D = \frac{\Delta u - \Delta s}{\Delta u + \Delta s - 2\Delta d}$.

Since $g_A^{np} = 1.2573 \pm 0.0028$ is by far the best measured parameter we will use this as a fix parameter and express the three other axial-vector form factors in terms of g_A^{np} and a_8 . This gives

$$\frac{g_A^{\Lambda p}}{\alpha(\Lambda)} = \frac{1}{6}a_8 + \frac{1}{2}g_A^{np}, \quad (24)$$

$$\frac{g_A^{\Sigma n}}{\alpha(\Sigma)} = \frac{1}{2}a_8 - \frac{1}{2}g_A^{np}, \quad (25)$$

$$\frac{g_A^{\Xi\Lambda}}{\alpha(\Xi)} = \frac{1}{3}a_8. \quad (26)$$

We have performed two least square fits of a_8 using these formulas and the experimental numbers quoted above, one with the α 's and one without. With the α 's we get $a_8 = 0.89 \pm 0.08$ with $\chi^2 = 0.31$ and without the α 's we get $a_8 = 0.70 \pm 0.08$ with $\chi^2 = 1.9$. We see that there is a considerable improvement when the α 's are included. These values correspond to $F/D = 0.75 \pm 0.04$ and $F/D = 0.63 \pm 0.04$ respectively.

We find a rather large deviation from the value $F/D = 0.575 \pm 0.016$ found by Close and Roberts [7], who deliberately have chosen not to include the induced form factor g_2 and use the value $g_A^{\Sigma n} = -0.340 \pm 0.017$ [4]. The deviation is also rather large relative to the value $a_8 = 0.601 \pm 0.038$ used by Ellis and Karliner [8] in their proton spin polarization analysis. The somewhat drastic increase in error comes from the unfortunately rather poor determination of the induced form factor g_2 .

Finally we remark that our evaluation of the isospin symmetry breaking parameter $T = \mu_u/\mu_d$ from the spin polarization [1] of the nucleon is not affected by this reinterpretation, since it is based on the ratio $\Delta\Sigma/g_A^{np}$ which is the same in both interpretations.

IV. IMPLICATIONS FOR THE PROTON SPIN POLARIZATION ANALYSIS

A change in the value of a_8 has a non-negligible influence on the proton spin polarization analysis. We will illustrate this using the formulas from the analysis of Ellis and Karliner

[8]. Their evaluation of $\Delta\Sigma = 0.31 \pm 0.07$ can be expressed as

$$\Delta\Sigma(Q^2) = 9 \frac{\Gamma_1^p(Q^2) - (\frac{g_A^{np}}{12} + \frac{a_8}{36})f(\alpha_s)}{h(\alpha_s)}, \quad (27)$$

where

$$f(\alpha_s) = 1 - \left(\frac{\alpha_s(Q^2)}{\pi}\right) - 3.5833 \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - 20.2153 \left(\frac{\alpha_s(Q^2)}{\pi}\right)^3 - \mathcal{O}(130) \left(\frac{\alpha_s(Q^2)}{\pi}\right)^4 \quad (28)$$

and

$$h(\alpha_s) = 1 - \left(\frac{\alpha_s(Q^2)}{\pi}\right) - 1.0959 \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2 - \mathcal{O}(6) \left(\frac{\alpha_s(Q^2)}{\pi}\right)^3. \quad (29)$$

The constant g_A^{np} has the usual value $g_A^{np} = 1.2573$, but the constant a_8 has in Ref. [8] the value $a_8 = 0.601 \pm 0.038$.

The value of $\Delta\Sigma$ will change appreciably if we change the value of a_8 . Let the change in a_8 be denoted δa_8 , and the new value of $\Delta\Sigma$ be denoted $\Delta\Sigma'$. We then have

$$\Delta\Sigma' = \Delta\Sigma - \frac{\delta a_8}{4} \left(1 - \mathcal{O}(2) \left(\frac{\alpha_s(Q^2)}{\pi}\right)^2\right) \approx \Delta\Sigma - \frac{\delta a_8}{4}. \quad (30)$$

The value of $a_8 = 0.89 \pm 0.08$ found above will thus lead to a different estimate of the total spin polarization of the proton. $\Delta\Sigma$ will change to

$$\Delta\Sigma' = 0.31 - \frac{0.29}{4} = 0.24 \pm 0.09. \quad (31)$$

In our previous analysis of isospin symmetry breaking in the baryon magnetic moments [1] this value favors a slightly smaller isospin symmetry breaking than the value $\Delta\Sigma = 0.31$. It also changes slightly the quark spin content of the proton to the values

$$\Delta u = 0.86 \pm 0.04, \quad (32)$$

$$\Delta d = -0.40 \pm 0.04, \quad (33)$$

$$\Delta s = -0.22 \pm 0.05. \quad (34)$$

calculated by means of $\Delta\Sigma'$, a_8 and g_A^{np} . The main effect is to allow Δs to be larger. This is consistent with the magnetic moment analysis in Ref. [1].

V. DISCUSSION AND CONCLUSIONS

As we have seen above there is supportive evidence from the axial-vector form factor data that the spin polarizations of the quarks are diminishing with the increase of mass of

the host particle. This mass dependence is born out in the sum-rules that can be written and are well satisfied by the experimental data.

One possible interpretation of this effect could be that the quarks simply lose their orientation as the excitation energy increases, and finally, at 3.6 GeV, become totally unoriented on the time scales considered here, much as if they were enclosed in a heat bath.

As the total angular momentum of the proton is fixed to 1/2, this means that there must be a contribution from some other electrically neutral component that increases its angular momentum with baryon mass to compensate for the decrease in the contribution coming from the quarks.

One possibility is to attribute such a contribution to the presence of gluonic components in the baryons. This is perfectly consistent with the findings from deep inelastic scattering experiments, that only about half of the proton momentum is carried by the quarks. Also a collective mode of the Skyrmin type, with a rather small contribution to the magnetic moment, could be envisaged to manifest in this way.

The new feature found here is that this contribution varies linearly with the mass of the baryon multiplet.

This could *e.g.* be the case if to this contribution there is associated a moment of inertia that grows with mass, but has stationary angular velocity.

As an *ad hoc* example we can consider a proton spin sum-rule of the form

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \frac{1}{2}I \cdot \text{const}, \quad (35)$$

where I is the moment of inertia of the non-quark component of the nucleon. When $I \propto m$, m being the baryon mass, we can rewrite this as

$$\Delta\Sigma(m) = \Delta\Sigma(m_p) - (m - m_p) \cdot c', \quad (36)$$

which leads to equation (9).

The puzzling outcome of this is that it predicts the quark spin polarization, and thus also the magnetic moment of a baryon, to vanish at $m \approx 3.6$ GeV, provided the linear relation between α and m is still valid there.

All this emphasizes the importance of trying to measure the magnetic moments of high mass baryon states and also to try to calculate them with lattice gauge techniques.

The studies performed by Leinweber *et al.* [9] supports indirectly the findings here. Their lattice gauge calculations have been done in quenched QCD by keeping the spin polarization fixed to the NQM values. The quark magnetic moments then show a decrease in value with increasing mass of the host particle, in much the same way as we found in Ref. [2], where we also chose to keep the spin polarizations mass independent.

Also lattice gauge calculations of the axial-vector form factors for the heavier states, would possibly shed light on the behavior found here. Such calculations have already been performed for the nucleon system [10].

Finally we have shown how a change in the evaluation of the axial-vector coupling constants will affect the proton spin polarization analysis. Our value for the constant a_8 favors a lower value of the proton quark spin sum.

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FIGURES

FIG. 1. The ratio $\alpha(B) = \mu_d^B / \mu_d^N$ as a function of the baryon mass. The points are the data for the nucleon, Λ , Σ and Ξ as given by equations (5)-(7). The straight line represents the linear fit according to equation (9).

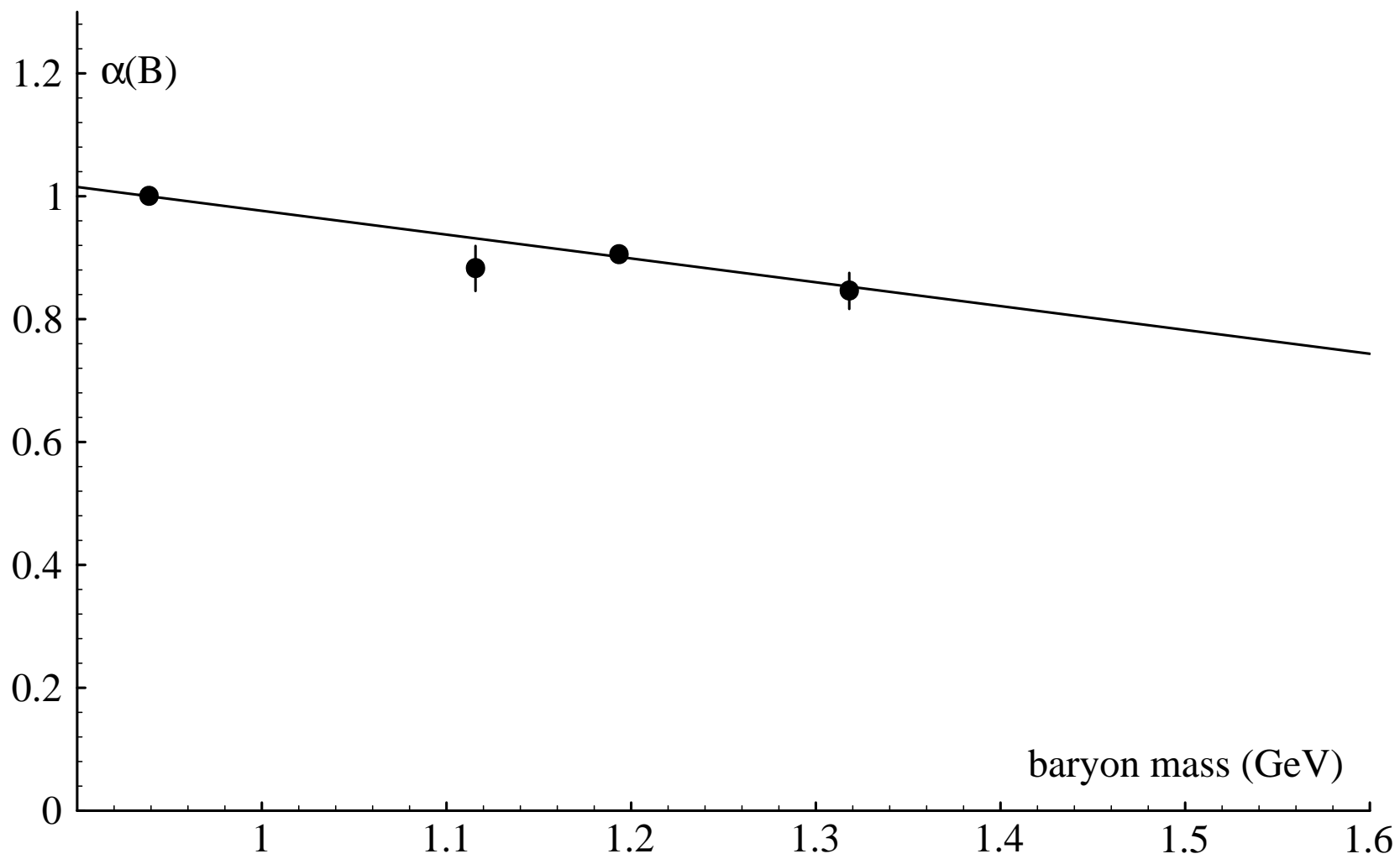


FIGURE 1